Instabilities and blowups in mathematical models of biological phenomena

Grzegorz Karch

University of Wrocaw, Poland
karch@math.uni.wroc.pl

Can biology inspire mathematics? By now, this is a rhetorical question because it is well known that mathematical models in biology generate several interesting problems which, to be solved, require new and highly nontrivial mathematical ideas. Here, biology inspires mathematicians in the same way as physics.

Can mathematicians offer anything to biology? Mathematical models of biological phenomena describe interactions between different elements of biological processes and allow us to deduce consequences of such interactions. Analytical and numerical studies of such mathematical models explain a nature of a biological process under consideration and allow to invent new experiments.

During my elementary talk, I shall illustrate such a research process concerning reciprocal interactions of biology and mathematics, by presenting results and ideas which were invented by Anna Marciniak-Czochra (Heidelberg University), Kanako Suzuki (Ibaraki University) and me, during several discussions in Heidelberg University, Tohoku University, and University of Wrocaw in years 2010–2013.

We studied a pattern formation arising in processes described by a system of a single reaction-diffusion equation coupled with ordinary differential equations. Such models were derived, for example, in studies of early cancerogenesis. We proved theorems which explain a mechanism of creations of unbounded and unstable patterns in such models.