On proper polynomial and holomorphic mappings

Zbigniew Jelonek

Polish Academy of Sciences, Poland
najelone@cyf-kr.edu.pl

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Let X, Y be smooth algebraic varieties of the same dimension. Let $f, g\colon X \longrightarrow Y$ be finite regular mappings. We say that f, g are equivalent if there exists a regular automorphism $\Phi \in \mathrm{Aut}(X)$ such that $f = g \circ \Phi$. Of course if f, g are equivalent, then they have the same discriminants (i.e., the same set of critical values) and the same geometric degree. We show, that conversely there is only a finite number of non-equivalent finite regular mappings $f\colon X \to Y$, such that the discriminant $D(f) = V$ and $\mu(f) = k$. As one of applications we show that if $f\colon \mathbb{C}^n \to \mathbb{C}^n$ is a proper mapping with $D(f) = \{x \in \mathbb{C}^n : x_1 = 0\}$, then f is equivalent to the mapping $g\colon \mathbb{C}^n \ni (x_1, \ldots, x_n) \mapsto (x_1^k, x_2, \ldots, x_n) \in \mathbb{C}^n$, where $k = \mu(f)$. Moreover, if $f\colon X \to Y$ is a finite mapping of topological degree two, then there exists a regular automorphism $\Phi\colon X \to X$ which acts transitively on fibers of f. In particular for $n > 1$ there is no finite mappings $f\colon \mathbb{P}^n \to \mathbb{P}^n$ of topological degree two.We prove the same statement in the local (and sometimes global) holomorphic situation. In particular we show that if $f\colon (\mathbb{C}^n, 0) \to (\mathbb{C}^n, 0)$ is a proper and holomorphic mapping of topological degree two, then there exist biholomorphisms $\Psi, \Phi\colon (\mathbb{C}^n, 0) \to (\mathbb{C}^n, 0)$ such that $\Psi \circ f \circ \Phi(x_1, x_1, \ldots, x_n) = (x_1^2, x_2, \ldots, x_n)$. Moreover, for every proper holomorphic mapping $f\colon (\mathbb{C}^n, 0) \to (\mathbb{C}^n, 0)$ with smooth discriminant there exist biholomorphisms $\Psi, \Phi\colon (\mathbb{C}^n, 0) \to (\mathbb{C}^n, 0)$ such that $\Psi \circ f \circ \Phi(x_1, x_1, \ldots, x_n) = (x_1^k, x_2, \ldots, x_n)$, where $k = \mu(f)$.