Fano manifolds whose elementary contractions are smooth $\mathbf{P}^1$-fibrations

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This presentation concerns a geometric characterization of complete flag varieties for semisimple algebraic groups. Namely, if X is a Fano manifold whose all elementary contractions are $\mathbf{P}^1$-fibrations then X is isomorphic to the complete flag manifold G/B where G is a semi-simple Lie algebraic group and B is a Borel subgroup of G.

Our proof of this statement is based on the following ideas: Every smooth $\mathbf{P}^1$-fibration of X provides an involution of the vector space $N^1(X)$ of classes of $\mathbf{R}$-divisors in X. We show that these involutions generate a finite reflection group, which is the Weyl group W of a semisimple Lie group G. Next we use $\mathbf{P}^1$-fibrations of X to define a set of auxiliary manifolds called Bott-Samelson varieties of X, which are analogues of the Bott-Samelson varieties that appear classically in the study of Schubert cycles of flag varieties. Subsequently we show that the recursive construction of appropriately chosen chain of Bott-Samelson varieties depends only on the combinatorics of the Weyl group W and ultimately we infer the isomorphism between X and G/B