

## On $p$ -adic Langlands program and geometry

**Przemysław Chojecki**

University of Oxford, England  
[chojecki@math.jussieu.fr](mailto:chojecki@math.jussieu.fr)

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Let  $l$  and  $p$  be two prime numbers. The Langlands program aims to establish a bijection with good properties between  $l$ -adic representations of absolute Galois groups of number fields (respectively,  $p$ -adic local fields; this is the local case) and  $l$ -adic representations attached to automorphic representations of reductive groups over number fields (respectively, certain smooth  $l$ -adic representations of reductive groups over  $p$ -adic fields). This domain of research has seen a spectacular progress during last two decades culminating in establishing the existence of the desired correspondence for  $GL_n$ , when  $l \neq p$  (work of Harris and Taylor).

The  $p$ -adic local Langlands correspondence focuses on the local case when  $l = p$ . This turns out to be much harder than the  $l \neq p$  case and demands different technical tools. As for now, the  $p$ -adic correspondence is known only in the case of  $GL_2(\mathbf{Q}_p)$  (by works of Berger, Breuil, Colmez, Emerton, Kisin, Paskunas and many others). There is a growing interest in generalizing this correspondence to other groups, especially because of many potential number-theoretic applications.

In our talk, we shall review recent progress in this theory, together with the latest technical input: completed cohomology of Emerton and perfectoid spaces of Scholze.