Tate sequences and Fitting ideals of Iwasawa modules

Cornelius Greither

Universität der Bundeswehr, Neubiberg, Germany
cornelius.greither@unibw.de

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Each number field L comes with its cyclotomic $\mathbf{Z}_p$-extension L_∞ (with p any fixed odd prime number), and to this one can associate a module X over the Iwasawa algebra. The so-called characteristic series already gives a lot of information on X (for instance it gives the λ-invariant). For a long time now the equivariant situation has been studied. Here L/k is a CM Galois extension, abelian in this talk for simplicity, and the characteristic series is replaced by a so-called Fitting ideal in the Iwasawa algebra $\Lambda = \mathbf{Z}_p[[Gal(L_\infty/k)]]$. After tensoring with $\mathbf{Q}$ (a process in which information is lost) and taking character parts, this gives back characteristic series. In a way the description of the Fitting ideal falls into two parts: the arithmetical part coming from p-adic L-functions, and the algebraic part, which gives certain "correcting" ideals by which one has to multiply the principal ideals generated by series associated to p-adic L-functions, in order to obtain the true Fitting ideal.

In this talk we will try to show how one can use the theory of Tate sequences to get a grip on the algebraic part of the problem. The arithmetic part depends very much on the specific extension L/k, and it is one of our main findings that the algebraic part only depends on the group $Gal(L/k)$. At the time being, we have to impose a restriction on L/k: we need that only places above p are ramified.

This is ongoing joint work with Kurihara, which also generalizes preceding work of Kurihara.