Galois representations attached to étale cohomology

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Session: 3. Arithmetic Geometry

Let K be a field and X/K a separated algebraic scheme. Grothendieck and Artin constructed for every prime number ℓ and every $i \in \mathbf{N}$ a cohomology group $H^i(X_{\overline{K}}, \mathbf{Q}_\ell)$ which comes with a natural action of the Galois group $\mathrm{Gal}(\overline{K}/K)$; one thus obtains a Galois representation

$$\rho_\ell\colon \mathrm{Gal}(\overline{K}/K) \to \mathrm{Aut}_{\mathbf{Q}_\ell}(H^i(X_{\overline{K}}, \mathbf{Q}_\ell)).$$

Denote by $\rho\colon \mathrm{Gal}(\overline{K}/K) \to \prod_\ell \mathrm{im}(\rho_\ell)$ the homomorphism induced by the ρ_ℓ. Serre proved recently that in the case where K is a number field the family $(\rho_\ell)_\ell$ is almost independent in the following sense: There exists a finite Galois extension K'/K such that

$$\rho(\mathrm{Gal}(\overline{K}/K')) = \prod_\ell \rho_\ell(\mathrm{Gal}(\overline{K}/K')).$$

This information is quite useful when working with such families of ℓ-adic representations attached to schemes, and it ties in well with the adelic openness conjecture. There are analogous results in the case where K is an arbitrary finitely generated field of characteristic zero, and where K is a geometric function field of arbitrary characteristic. Following a suggestion of Illusie and making strong use of results of Orgogozo this was used to establish a quite general independence theorem for families $(F_\ell)_\ell$ of étale sheaves of $\mathbf{F}_\ell$-vector spaces over an arithmetic scheme which satisfy a uniform constructability and a potential semistability condition.