

Carleson measures and composition operators on abstract Hardy spaces

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In this talk I will report on some recent results about abstract Hardy spaces obtained in collaboration with Mieczysław Mastyło (Adam Mickiewicz University Poznań, Poland).

Let \mathbb{D} be the open unit disk of the complex plane and the torus $\mathbb{T} = \partial\mathbb{D}$ its border equipped with the normalized length measure m . Essentially in the same way that Hardy spaces $H^p(\mathbb{D})$ are defined out of Lebesgue spaces $L^p(\mathbb{T})$, one can define an abstract Hardy space HX out of any (quasi-)Banach function space X on the torus \mathbb{T} :

$$HX := \left\{ f \in H(\mathbb{D}); \|f\|_{HX} := \sup_{0 \leq r < 1} \|f_r\|_X < +\infty \right\}.$$

We will consider mainly the case where X is a symmetric (that is, rearrangement invariant) Banach space on (\mathbb{T}, m) . Particular classes of abstract Hardy spaces are Hardy-Orlicz, Hardy-Lorentz and Hardy-Marcinkiewicz spaces, which correspond respectively to the cases where X is an Orlicz, a Lorentz or a Marcinkiewicz space on (\mathbb{T}, m) . In these cases we have $HX = \{f \in H^1(\mathbb{D}) : \tilde{f} \in X\}$, where \tilde{f} is the boundary value of f (the radial limits).

Motivated by the study of composition operators on HX we study the inclusion of HX into a Banach symmetric space $Y(\mu)$, for μ a finite measure on \mathbb{D} . We investigate the relationships between boundedness or compactness of the inclusion of HX in $Y(\mu)$ and some conditions on μ which are variants of the requirement for μ to be a Carleson measure. An important role in the definition of these conditions is played by the fundamental functions of X and Y . In particular we will give a characterization of the compactness of composition operators on Hardy-Lorentz and Hardy-Marcinkiewicz spaces. This characterization is similar to the one given for Hardy-Orlicz spaces by Lefèvre, Li, Queffélec and Rodríguez-Piazza [1].

References

- [1] P. Lefèvre, D. Li, H. Queffélec and L. Rodríguez-Piazza, *Composition operators on Hardy Orlicz spaces*, *Memoirs of the American Mathematical Society*, Vol. 207, American Mathematical Society, Providence, RI, 2010.