Asymptotic estimates on an inequality of von Neumann for homogeneous polynomials

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The talk is based on the joint work with D. Galicer and S. Muro

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We consider the following m-homogeneous version of an inequality of von Neumann: there exists a positive constant $C_{k,q}(n)$ such that for every k-homogeneous polynomial p in n variables and every n-tuple of commuting operators $(T_1, \ldots, T_n)$ with $\sum_{i=1}^{n} \|T_i\|^q \leq 1$ we have

$$\|p(T_1, \ldots, T_n)\|_{\mathcal{L}(\mathcal{H})} \leq C_{k,q}(n) \ \sup\{|p(z_1, \ldots, z_n)| : \textstyle\sum_{i=1}^{n} |z_i|^q \leq 1\}\,.$$

A long standing problem is, for fixed k and q, to study the asymptotic growth of the smallest constant $C_{k,p}(n)$ as n (the number of variables/operators) tends to infinity.

Dixon for $q = \infty$ [1] and Mantero and Tongue for $1 \leq q < \infty$ [2] gave upper and lower bounds for $C_{k,p}(n)$. We go on with this study, showing that the upper bound given by Dixon is optimal and improving the lower bound given by Mantero and Tonge for $2 \leq q < \infty$.

References

[1] P. G. Dixon, *The von Neumann inequality for polynomials of degree greater than two*, London Math. Soc, 14, 1976, 369–375.

[2] A. M. Mantero, and A. Tonge, *Banach algebras and von Neumann's inequality*, Proceedings of the London Mathematical Society, 3(2), 1979, 309–334.