On the space of maximal ideals of vector lattices

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If the set $\mathfrak{M}(E)$ of all maximal ideals of an Archimedean vector lattice E is equipped with the hull-kernel topology τ_{hk} then the topological space $\big(\mathfrak{M}(E), \tau_{hk}\big) =: \mathfrak{M}(E)$ turns out to be a Hausdorff space but, in general, does not satisfy any stronger separation axiom. The space $\mathfrak{M}(E)$ carries many information on the vector lattice E.

An element $\varphi \in E$ is called *finite* if there exists an element $z \in E$ (a majorant of φ) satisfying the property: for each $x \in E$ there is a number $c_x > 0$ such that the inequality

$$|x| \wedge n|\varphi| \leq c_x z$$

holds for any $n \in \mathbb{N}$. The element φ is called *totally finite*, if it possesses a majorant which itself is a finite element. A finite element φ is called *selfmajorizing*, if $|\varphi|$ is a majorant of φ.

Finite, totally finite and selfmajorizing elements of a vector lattice E can be characterized by means of the subsets $G_x := \{M \in \mathfrak{M}(E) : x \notin M\}$ which are defined in $\mathfrak{M}(E)$ for any $x \in E$. The τ_{hk}-closure $\operatorname{supp}_{\mathfrak{M}}(x)$ of G_x is the so-called *abstract support* of the element x. In radical-free vector lattices a finite element φ is characterized by the compactness of its abstract support and the majorants z of φ by the inclusion $\operatorname{supp}_{\mathfrak{M}}(\varphi) \subset G_z$. Totally finite and selfmajorizing elements can be similarly characterized.

The space $\mathfrak{M}(E)$ plays an important role for the representation of vector lattices by means of continuous functions, namely in the situation if one asks for representations, where the isomorphic images of finite elements are functions with compact support.

References

[1] M. R. Weber, *Finite Elements in Vector Lattices.* W. de Gruyter, Berlin/Boston, 2014.