Spectral properties of the $\overline{\partial}$-Neumann operator

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We consider the $\overline{\partial}$-Neumann operator

$$N : L^2_{(0,q)}(\Omega) \longrightarrow L^2_{(0,q)}(\Omega),$$

where $\Omega \subset \mathbb{C}^n$ is bounded pseudoconvex domain, and

$$N_\varphi : L^2_{(0,q)}(\Omega, e^{-\varphi}) \longrightarrow L^2_{(0,q)}(\Omega, e^{-\varphi}),$$

where $\Omega \subseteq \mathbb{C}^n$ is a pseudoconvex domain and φ is a plurisubharmonic weight function. N is the inverse to the complex Laplacian $\Box = \overline{\partial}\,\overline{\partial}^* + \overline{\partial}^*\,\overline{\partial}$.

Using a general description of precompact subsets in L^2-spaces we obtain a characterization of compactness of the $\overline{\partial}$-Neumann operator, which can be applied to related questions about Schrdinger operators with magnetic field and Pauli and Dirac operators and to the complex Witten Laplacian. In this connection it is important to know whether the Fock space

$$\mathcal{A}^2(\mathbb{C}^n, e^{-\varphi}) = \{f \colon \mathbb{C}^n \longrightarrow \mathbb{C} \text{ entire} : \int_{\mathbb{C}^n} |f|^2 e^{-\varphi}\, d\lambda < \infty\}$$

is infinite-dimensional, which depends on the behavior at infinity of the eigenvalues of the Levi matrix of the weight function φ.

In addition we discuss obstructions to compactness of the $\overline{\partial}$-Neumann operator, and we describe, in some special cases, the spectrum of the $\Box$-operator.

References

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