Lifting maps to the spectral ball

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The spectral unit ball Ω_n is the set of all matrices $M \in \mathbb{C}^{n\times n}$ with spectral radius less than 1. Let us call π the "projection" map which to a matrix M associates $\pi(M) \in \mathbb{C}^n$, the coefficients of its characteristic polynomial (essentially), in fact the elementary symmetric functions of its eigenvalues. Let $\mathbb{G}_n := \pi(\Omega_n)$.

When investigating Pick-Nevanlinna problems for maps from the disk to Ω_n, it is often useful to project the map to the symmetrized polydisk (for instance to obtain continuity results for the Lempert function, related to the two-point problem): if $\psi \in \mathcal{O}(\mathbb{D}, \Omega_n)$ and $\psi(\alpha_j) = M_j$, $1 \le j \le N$, then $\pi \circ \psi \in \mathcal{O}(\mathbb{D}, \mathbb{G}_n)$ and $\pi \circ \psi(\alpha_j) = \pi(M_j)$, $1 \le j \le N$. Given a map $\varphi \in \mathcal{O}(\mathbb{D}, \mathbb{G}_n)$, we are looking for necessary and sufficient conditions for this map to "lift through given matrices", i.e. find ψ as above so that $\pi \circ \psi = \varphi$. This is problematic when the matrices M_j are derogatory (i.e. do not admit a cyclic vector). There are natural necessary conditions, involving not only the values: $\varphi(\alpha_j) = \pi(M_j)$, of course, but also derivatives of φ at the points α_j. Those conditions turn out to be sufficient in small dimensions (up to 4).