

## Spectra of formulae with restrictions

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Given a formula  $\phi$  of some logic, the **spectrum** of  $\phi$ , denoted  $\text{spec}(\phi)$ , is the set of non-negative integers  $n$  such that  $\phi$  has a model of cardinality  $n$ . The notion of a spectrum has been introduced by Scholz in 1952, along with the main problem: characterize sets of integers which appear as spectra of a given class of formulae. Fagin has shown in his seminal paper in 1974 that a set  $S$  is a spectrum of some first order formula  $\phi$  iff the set of unary representations of elements of  $S$  is in **NP**. This also shows that the main open problem from spectra theory, called Asser's conjecture – whether a complement of a spectrum is also a spectrum – is equivalent to the complexity theoretic problem **NEXPTIME=co-NEXPTIME**. An interesting question is, what happens when we put restrictions on the vocabulary, formulae, or models involved? We are going to talk about the following restrictions:

- We consider only models of bounded degree (to be more precise, models whose Gaifman graphs are of bounded degree). In this case, we provide lower and upper complexity theoretic bounds for spectra of this kind, with only a polylogarithmic gap.
- We consider only models whose Gaifman graphs are planar. Again, we provide lower and upper complexity theoretic bounds for spectra of this kind, and the gap is polynomial. We can also consider only formulae whose all models are planar graphs. In this case, the gap is bigger (the lower bound uses just logarithmic memory).
- We consider only formulae of  $C^2$  logic, that is, using only two variables and counting up to some threshold (“there exist at least  $k$  elements  $x$  such that  $\phi(x, y)$ ”). In this case, we show that spectra are exactly ultimately periodic sets, and in the more general multi-dimensional case where we count the number of elements satisfying predicate  $P_i$  for  $i = 1, \dots, k$ , they are exactly semilinear sets. This shows that Asser's conjecture holds for  $C^2$  logic.
- Let  $\text{spec}_k$  be spectra of formulae of  $FO_k$  logic (using  $k$  variables). We show that for  $k \geq 2$   $\text{spec}_k \subsetneq \text{spec}_{2k+2}$ , hence the variable hierarchy does

not collapse.