

Global Realisations of Local Specifications

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The following situation is typical of several tasks of model construction that arise, e.g., in connection with finite model properties and finite controllability. Given a finite set of *local templates*, i.e., isomorphism types of finite structures, together with specifications of required and permitted pairwise *overlaps* between them, the task is to find finite global *realisations*. In the most general sense, a *realisation* is a single structure that is the union of a family \mathcal{S} of designated substructures, each isomorphic to one of the given templates and with pairwise overlaps in accordance with the specification. We think of the overlap specification as prescribing required and permitted pairwise overlaps, specified by *partial isomorphisms* of the form $\rho: V_s \rightarrow V_t$ between templates V_s and V_t , in the following sense:

- for each specified overlap between templates V_s and V_t , every instance of V_s in \mathcal{S} overlaps with some instance of V_t in \mathcal{S} as prescribed by ρ ;
- there are no incidental overlaps: any non-trivial intersection between two substructures in \mathcal{S} conforms to some composition $\rho_n \circ \dots \circ \rho_1$ of prescribed overlaps.

We approach the finite realisation task via a generic construction of reduced products with suitable groupoids. Due to strong acyclicity properties, these groupoids can serve as structural backbones in the construction of finite realisations, similar to the use of free groups in standard constructions of infinite realisations. The underlying groupoids are constructed in a process that interleaves model-theoretic amalgamation techniques with groupoidal analogues of permutation group actions. The proposed constructions are sufficiently generic and natural as to be compatible with symmetries of the given specification while also allowing us to avoid incidental cycles of overlaps up to any specified finite length. It follows that the resulting finite realisations can be forced

- to realise certain symmetries built into the specification, and
- to admit local, size-bounded homomorphisms into any (finite or infinite) realisation and in particular into the (infinite) canonical free realisation.

As one remarkable consequence we obtain a new proof of the powerful finite-model-assertion in a theorem of Herwig and Lascar, which allows us to lift

partial isomorphisms (local symmetries) of finite structures to automorphisms (global symmetries) within classes defined by forbidden homomorphisms.