Behaviour of solutions of one neuron models

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The talk is based on the joint work with Aija Anisimova and Maruta Avotina.

Session: 7. Difference equations and their application in the mathematical modeling

Neural networks are complex and large-scale nonlinear dynamical systems. As the proverbial forest can not be seen through because of the trees, a detailed study of single neuron is an interesting subject itself, but it is not necessary to understand the macroscopic dynamics and role of neural networks. In the literature [2] a delay differential equation

$$x'(t) = -g(x(t-\tau)) \qquad (1)$$

is used as a model for a single neuron with no internal decay where $g : \mathbf{R} \to \mathbf{R}$ is either a sigmoid or a piecewise linear signal function and $\tau \leq 0$ is a synaptic transmission delay. From equation (1) we obtain a difference equation

$$x_{n+1} = \beta x_n - g(x_n). \qquad (2)$$

By [2] x denotes the activation level of a neuron, β is interpreted as an internal decay rate and g is a signal function. Accordingly to the parameter β we obtain different behaviour of solutions of difference equation (1). Idea of finding periodic orbits of the model first was demonstrated in [3]. Signal function play an important role in the investigation. In our work we used step functions with two and three thresholds (see [1]) therefore in fact we investigated one dimensional discontinuous piecewise linear map. We will present some results about the solutions of model (2) with different signal functions.

References

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[3] Z.Zhou, *Periodic Orbits on Discrete Dynamical Systems*, Computers and Mathematics with Applications 45, 2003, 1155-1161.