On asymptotic equivalence of difference equations

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Consider the following system of difference equations in Banach space $\mathbf{X} \times \mathbf{Y}$:

$$\begin{cases} x(t+1) &= A(t)x(t) + f(t, x(t), y(t)), \\ y(t+1) &= B(t)y(t) + g(t, x(t), y(t)), \end{cases} \tag{1}$$

satisfying the *conditions of separation*

$$\nu = \max\left(\sup_{t\in\mathbf{Z}} \sum_{s=-\infty}^{t-1} |Y(t, s+1)|\,|X(s,t)|, \sup_{t\in\mathbf{Z}} \sum_{s=t}^{+\infty} |X(t, s+1)|\,|Y(s,t)|\right) < +\infty$$

and $f(t,\cdot)$, $g(t,\cdot)$ are ε-Lipshitz, $f(t,0,0) = 0$, $g(t,0,0) = 0$.

We find a simpler system of difference equations that is conjugated and asymptotic equivalent to the given one. Using this result we obtain sufficient conditions that invertible and noninvertible system (1) is asymptotic equivalent to the linear system

$$\begin{cases} x(t+1) &= A(t)x(t), \\ y(t+1) &= 0, \end{cases} \tag{2}$$

in the case when ε depends on t and tends to zero as $t \to +\infty$ sufficiently rapidly.

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