Numerical solution of the 1D subdiffusion equation with two moving boundaries

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Moving boundary problems are a special case of the boundary value problems. They are often called Stefan problems and were extensively studied in the partial differential equations theory (compare monograph [1] and the references therein). The description including moving boundaries was applied in modeling of the formation of sedimentary ocean deltas [2] and the moisture transport such as swelling grains or polymers [3]. The fractional extension of the dual moving boundaries problem is used as the mathematical model of a drug release from a polymeric matrix [4]. We shall construct a numerical solution of the system of equations presented below:

$$ {}^{c}D^{\alpha}_{0+,\tau} f(X,\tau) = \frac{\partial^2 f(X,\tau)}{\partial X^2}, \quad S_1(\tau) < X < S_2(\tau), \quad \tau > 0 \tag{1} $$

$$ f(S_1(\tau),\tau) = 1, \quad f(S_2(\tau),\tau) = 0, \quad \tau > 0 \tag{2} $$

$$ f(0^+,0) = 0, \quad S_1(0) = 0, \quad S_2(0) = 0 \tag{3} $$

$$ {}^{c}D^{\alpha}_{0+,\tau} S_2(\tau) = -\Lambda_2 \frac{\partial f(X,\tau)}{\partial X}|_{X=S_2(\tau)} \tag{4} $$

$$ {}^{c}D^{\alpha}_{0+,\tau} S_1(\tau) = \Lambda_1 \frac{\partial f(X,\tau)}{\partial X}|_{X=S_1(\tau)} \tag{5} $$

which constitute the 1D fractional Stefan problem with two moving boundaries given as $S_1(\tau)$ and $S_2(\tau)$. In our approach we use new spatial variable $u = \frac{X-S_1(\tau)}{S_2(\tau)-S_1(\tau)}$ as in this new coordinates system: (u,τ) the boundaries are fixed.

References

[1] S.C. Gupta. *The Classical Stefan Problem. Basic Concepts, Modeling and Analysis.* Elsevier, Amsterdam, 2003.

[2] J. Lorenzo-Trueba, V.R. Voller. *Analytical and numerical solution of a generalized Stefan problem exhibiting two moving boundaries with application to ocean delta formation.* J. Math. Anal. Appl. 366, 2010, 538–549.

[3] S.I. Barry, J. Caunce. *Exact and numerical solutions to a Stefan problem with two moving boundaries.* Appl. Math. Model. 32, 2008, 83–98.

[4] Chen Yin, Mingyu Xu *An asymptotic analytical solution to the problem of two moving boundaries with fractional diffusion in one-dimensional drug release devices.* J. Phys. A: Math. Theor. 42, 2009, 115210.