Numerical solution of the fractional Euler-Lagrange equation with natural boundary conditions

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The fractional Euler-Lagrange equations contain the left and right derivatives. It is an additional drawback concerning the computation of an exact solution. Therefore, in this paper we propose a numerical solution of the fractional Euler-Lagrange equation in the finite time interval $t \in [0, b]$ in form

$$-{}^{C}D^{\alpha}_{b^-}\, D^{\alpha}_{0+}\, f(t) + \omega^2 f(t) = g(t), \tag{1}$$

with natural boundary conditions

$$f(0) = 0, \quad D^{\alpha}_{0+} f(t)\big|_{t=b} = 0. \tag{2}$$

Next we present the discrete form of Eq. (1)

$$\sum_{k=i}^{n} \left[v(n-i, n-k) \sum_{j=0}^{k} v(k,j)\; f_j \right] - \omega^2 f_i = -g_i, \;\; \text{for } i = 1, ..., n-1. \tag{3}$$

Finally, we present examples of numerical solutions of Eq. (1) (see Figure 1)

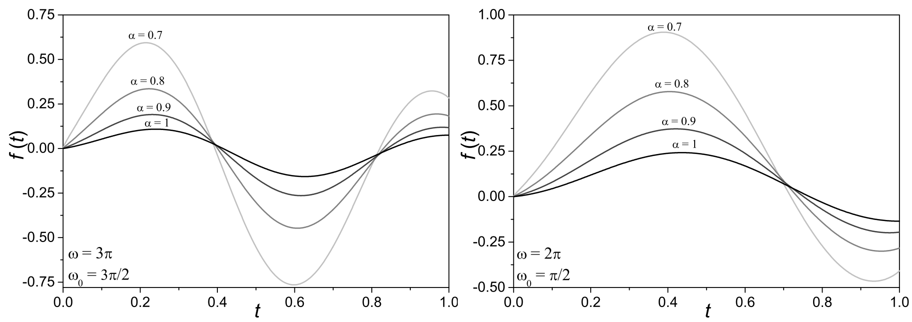

Figure 1: Examples of numerical solutions of Eq. (1) for $g(t) = 5\cos(\omega_0 t)$