A global implicit function theorem with applications to fractional problems

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The main result of the paper is the following global implicit function theorem.

Theorem *Let X, Y be real Banach spaces, H - a real Hilbert space. If $F : X \times Y \to H$ is continuously differentiable on $X \times Y$ and*

- *differential $F'_x(x,y) : X \to H$ is bijective for any $(x,y) \in X \times Y$*
- *for any fixed $y \in Y$, the functional*

$$\varphi : X \ni x \longmapsto (1/2)\,\|F(x,y)\|^2 \in \mathbb{R}$$

satisfies the Palais-Smale condition,

then there exists a unique function $\lambda : Y \to X$ such that $F(\lambda(y), y) = 0$ for any $y \in Y$ and this function is continuously differentiable on Y with differential $\lambda'(y)$ at $y \in Y$ given by

$$\lambda'(y) = -[F_x(\lambda(y), y)]^{-1} \circ F_y(\lambda(y), y).$$

Some applications of the theorem to problems containing the integrals and derivatives of fractional order are given.