Regular Sturm-Liouville problem with Riemann-Liouville derivatives of order $\alpha \in (1, 2)$

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We consider a regular fractional Sturm-Liouville problem (FSLP):

$$\mathcal{L}y(x) = D^{\alpha}_{b-}p(x)D^{\alpha}_{a+}y(x) = \lambda y(x) \tag{1}$$

$$y(a) = 0 \quad y(b) = 0, \tag{2}$$

$$y'(a) = 0 \quad D^{\alpha}_{a+}y(x)|_{x=b} = 0, \tag{3}$$

where order of Riemann-Liouville derivatives $\alpha \in (1, 2)$ and p is an arbitrary positive function from the $C[a, b]$-space. The Riemann-Liouville derivatives, respectively the left and the right, are the following integro-differential operators

$$D^{\alpha}_{a+}f(x) := \frac{1}{\Gamma(2-\alpha)}\frac{d^2}{dx^2}\int_a^x (x-v)^{1-\alpha}f(v)\, dv$$

$$D^{\alpha}_{b-}f(x) := \frac{1}{\Gamma(2-\alpha)}\frac{d^2}{dx^2}\int_x^b (v-x)^{1-\alpha}f(v)\, dv.$$

Using the inverse integral operator approach we prove the theorem below.

Theorem 1. *Fractional Sturm-Liouville problem* (1)–(3) *has an infinite countable set of positive, simple eigenvalues: $\Lambda_1 < \Lambda_2 < ....$ and the corresponding orthonormal set of differentiable eigenfunctions is a basis in the $L^2(a, b)$ - space, provided $\frac{3}{2} < \alpha < 2$ and $p \in C[a, b]$ is an arbitrary positive function.*

The obtained result extends methods of solving such problems, known from the classical Sturm-Liouville theory, as well as recent results for problems with derivatives of fractional order $\alpha \in (0, 1)$ [1]. It also is an alternative approach to variational methods, effective in the case of FSLP with Caputo fractional derivatives [2].

References

[1] M. Klimek and M. Błasik,*Regular Fractional Sturm-Liouville problem with discrete spectrum: solutions and applications.* To be published in: Proceedings of the 2014 International Conference on Fractional Differentiation and Its Applications, Catania 23-25 June, Italy.

[2] M. Klimek, T. Odzijewicz and A. Malinowska, *Variational methods for the fractional Sturm-Liouville problems*, J. Math. Anal. Appl. 416, 2014, 402–426. dx.doi.org/10.1016/j.jmaa.2014.02.009.