Existence of optimal solution to some Bolza problem governed by Dirichlet fractional problem[1]

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We investigate the problem of the existence and continuous dependence of solution to the following Dirichlet problem

$$\begin{cases} f_{x_1}(t,x,D^{\alpha}_{a+}x,y,D^{\beta}_{a+}y,u) = D^{\alpha}_{b-}f_{x_2}(t,x,D^{\alpha}_{a+}x,y,D^{\beta}_{a+}y,u) \\ f_{y_1}(t,x,D^{\alpha}_{a+}x,y,D^{\beta}_{a+}y,u) = D^{\beta}_{b-}f_{y_2}(t,x,D^{\alpha}_{a+}x,y,D^{\beta}_{a+}y,u) \end{cases} \tag{1}$$

$$\begin{cases} I^{1-\alpha}_{a+}x\,(a) = x\,(a) = x\,(b) = 0, \\ I^{1-\beta}_{a+}y\,(a) = y\,(a) = y\,(b) = 0, \end{cases} \tag{2}$$

Since the assumptions we made does not guarantee the uniqueness of solution to (1)–(2) we use the notion of Kuratowski–Painlevé limit to describe the mentioned continuous dependence.

Applying continuous dependence we also prove theorem on existence of optimal solution to the following Bolza problem:

(B) minimize

$$\mathcal{B}\,(u,x_u,y_u) := \int_a^b B_1(t,x_u\,(t)\,,D^{\alpha}_{a+}x_u\,(t)\,,y_u\,(t)\,,D^{\beta}_{a+}y_u\,(t)\,,u\,(t))dt \\ + B_2\,(x_u\,(T)\,,y_u\,(T))$$

where $(x_u\,(.)\,,y_u\,(.))$ is any solution to (1)–(2) corresponding to $u\,(.) \in \mathcal{U}_L$, $T := \frac{b-a}{2}$,

$$\mathcal{U}_L := \{u\,(.) \in L^{\infty}\,([a,b],M) : |u\,(t_1) - u\,(t_2)| \le L\,|t_1 - t_2| \\ \text{for a.e. } t_1,t_2 \in [a,b]\}$$

and $M \subset \mathbb{R}^m$ is a given convex and compact set.

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