Helmholtz theorem for Hamiltonian systems on time scales

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The talk is based on the joint work with Jacky Cresson

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A classical problem in Analysis is the well-known *Helmholtz's inverse problem of the calculus of variations*: find a necessary and sufficient condition under which a (system of) differential equation(s) can be written as an Euler-Lagrange or a Hamiltonian equation and, in the affirmative case, find all the possible Lagrangian or Hamiltonian formulations. This condition is usually called *Helmholtz condition*. Generalisation of this problem in the discrete calculus of variations framework has been done in [2] and [7] in the discrete Lagrangian case. For the Hamiltonian case it has been done for discrete calculus of variation in [5] using the framework of [6] and in [4] using a discrete embedding procedure. In this talk we will generalized the Helmholtz theorem for Hamiltonian systems in the case of time-scale calculus using the work of [3] and [1].

References

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