Topological Fractals and Multifractals: their properties and metrization

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In this talk we shall discuss some topological and metric properties of topological fractals and multifractals.

A topological space X is called a *topological fractal* if $X = \bigcup_{f \in \mathcal{F}} f(X)$ for a finite family $\mathcal{F}$ of continuous self-maps of X such that for every open cover $\mathcal{U}$ of X there is a number n such that for every choice of maps $f_1, \dots, f_n \in \mathcal{F}$ the set $f_1 \circ \dots \circ f_n(X)$ is contained in some set $U \in \mathcal{U}$. We shall prove that each Hausdorff topological fractal is compact and metrizable. Moreover, its topology is generated by a metric d making all maps $f \in \mathcal{F}$ Edelstein contractive in the sense that $d(f(x), f(y)) < d(x, y)$ for any distinct points $x, y \in X$.

Topological fractals are partial cases of multifractals. A topological space X is called a *multifractal* if there is a continuous finitely-valued map $\Phi : X \multimap X$ such that $X = \lim_{n\to\infty} \Phi^n(x)$ for every point $x \in X$. The class of multifractals is much wider than the class of topological fractals. For example, each compact Hausdorff space admitting a minimal action of a finitely generated group is a multifractal. This implies that there are compact connected multifractals which are not locally connected, there is a first countable compact multifractal, which is not metrizable, etc.