Hyperspaces — where fractals live

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As fractals historically are generated by iterated function systems on a space X, each IFS determines a (continuous) self-map of the assigned hyperspace $K(X)$ of compact subsets of X, for instance. Their fixed points are commonly viewed as hot candidates to be called "fractals".

So, it seems to be natural to consider not exclusively such self-mappings of the hyperspace, that are generated from IFS, but to consider first arbitrary continuous self-mappings and then to examine under which conditions they have (nice) fixed points.

The notion "fractal" may come from "non-integer dimension", but to stay with this understanding of a fractal object, requires to have a suitable notion of dimension. Unfortunately, this is not always present in more general contexts than metric or friendly topological spaces.

Although it may be an interesting research area to extend the notion of dimension to wider classes of spaces in an appropriate manner, we propose here another idea: to study the fixed points of continuous self-mappings of hyperspaces.

To do this categorically, we need primarily a functional notion of hyperspace.

Here we will propose and discuss such an approach to define hyperstructures, which works in every cartesian closed topological category, and so applies to every topological category, using it's topological universe hull.