Demyanov Difference in infinite dimensional Spaces

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We generalize the Demyanov difference to the case of real Hausdorff topological vector spaces.

For $A, B \subset X$ we define *upper difference* $\mathcal{E}_{A,B}$ as the family $\mathcal{E}_{A,B} = \{C \in \mathcal{C}(X) | A \subset \overline{B + C}\}$, where $\mathcal{C}(X)$ is the family of all nonempty closed convex subsets of the topological vector space X. We denote the family of inclusion minimal elements of $\mathcal{E}_{A,B}$ by $m\mathcal{E}_{A,B}$. We define a new subtraction by $A \overset{D}{-} B = \overline{\text{conv}} \bigcup m\mathcal{E}_{A,B}$. We show that $A \overset{D}{-} B$ is a generalization of Demyanov difference. We prove some clasical properties of the Demyanov difference. For a locally convex vector space X and compact sets $A, B, C \in \mathcal{C}(X)$ the Demyanov-Difference has the following properties:

(D1) If $A = B + C$, then $C = A \overset{D}{-} B$.

(D2) $(A \overset{D}{-} B) + B \supset A$.

(D3) If $B \subset A$, then $0 \in A \overset{D}{-} B$.

(D4) $(A \overset{D}{-} B) = -(B \overset{D}{-} A)$

(D5) $A \overset{D}{-} C \subset (A \overset{D}{-} B) + (B \overset{D}{-} C)$.

In the proofs we use a new technique which is based on the following lemma. *Let X be a Hausdorff topological vector space, A be closed convex, B bounded subset of X. Then for every bounded subset M we have* $\overline{A + M} = \bigcap_{C \in \mathcal{E}_{A,B}} \overline{B + C + M}$.

We also give connections between Minkowski subtraction and the union of upper differences.

Let X be a Hausdorff topological vector space, A be closed convex, B bounded subset of X. Then $A \dot{-} B = \bigcap \mathcal{E}_{A,B}$ *where* $A \dot{-} B = \{x \in X | B + x \subset A\}$.

We show that in the case of normed spaces the Demyanov difference coincides with classical definitions of Demyanov subtraction.

References

[1] Grzybowski, J., Pallaschke, D., and Urbanski, R.; *Demyanov difference in infinite dimensional spaces*, Constructive Nonsmooth Analysis and Related Topics, Springer Optimization and Its Applications 87 (2014), 13-24.