

Minimax theorem with joint Φ -convexlike property

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Φ -convexity, a form of abstract convexity, was first introduced by Ky Fan [2] and next investigated by Pallaschke and Rolewicz [3], Rubinov [4], Singer [6] and many other authors.

The present talk is devoted to minimax theorems for Φ -convex functions.

Starting from the paper by Ky Fan [1] convexlike properties were used in those minimax theorems which do not refer to linear structures of the underlying spaces.

Let X be a set and Φ be a class of functions $\varphi : X \rightarrow \mathbb{R}$. Following Ky Fan [1] we say that the class Φ is *convexlike on X* if for any $x_1, x_2 \in X$ and $t \in [0, 1]$ there exists $x_0 \in X$ such that

$$\varphi(x_0) \leq t\varphi(x_1) + (1-t)\varphi(x_2) \quad \text{for } \varphi \in \Phi.$$

Numerous extensions or generalizations of convexlikeness have been proposed (see for example [1], [5]).

We introduce joint convexlikeness which generalizes the convexlikeness and is shaped for Φ -convex functions.

The property of joint Φ -convexlikeness allows us to obtain minimax theorem for functions with not necessarily connected level sets.

References

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