Vector spaces of convex sets, representation of its elements and application to crystal growth description

Ryszard Urbański

Adam Mickiewicz University, Poznań, Poland
University of Karlsruhe, Karlsruhe, Germany
rich@amu.edu.pl, jgrz@amu.edu.pl, diethard.pallaschke@kit.edu

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We have analyzed quasidifferential [1] (gradient of a wide class of non-smooth functions) and observed that it is not unique and often very large and difficult to handle. We set our objective to lay theoretical foundations for finding the best or smallest possible representation of quasidifferential. For this purpose we investigated Minkowski addition of convex sets, an inverse operation of subtraction and a quotient vector space of differences of sets.

We found out that minimal representation (minimal pair) exists for compact sets [3]. We discovered that minimal pair is unique in two dimensions and, in general, it is not unique in three dimensions [3]. We also obtaind significant results concerning minimal representation of the difference of polygons and polyhedra, certain criteria of minimality and methods of reduction [3].

We conclude that foundations for theory of minimal pairs are laid. However, further studies are needed. In particular there is need to determine existence of minimal pairs in non-reflexive Banach spaces, to find necessary and sufficient condition of minimality in three dimensions and to improve methods of reduction. We also need to investigate possible areas of implementation of vector spaces of convex sets and minimal representations of its elements. An example of such areas is crystal growing. In particular we were able to give a formula of crystal growth if faces of a crystal grow with constant velocity and to state necessary and sufficient condition for crystal face disappearing [2].

References

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