A sense preseving homeomorphism with a.e. negative Jacobian determinant

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The talk is based on a joint work with Piotr Hajłasz

Session: 11. Geometric Analysis and Related Topics

A standard assumption in the geometric theory of deformations is that the mappings that are the object of study have non-negative Jacobian determinant. This is rather obvious when the deformations in question are diffeomorphisms (the condition that the deformation preserves sense). A natural question arises: how much topological information does this condition carry for weakly differentiable homeomorphisms? One can ask a more specific question: does a sense preserving, weakly differentiable homeomorphism (of an n-dimensional cube) necessarily have non-negative Jacobian? A positive answer has been given recently (by Hencl and Maly) for $W^{1,p}$ homeomorphisms with $p > n/2$ (and for $p = 1, 2, n = 3$). It has been conjectured that a likewise answer holds for all $W^{1,1}$ homeomorphisms.

In a joint work with Piotr Hajłasz (University of Pittsburgh), we provide sort of a counterexample to the conjecture, although the homeomorphism of the n-dimensional cube we construct is not in $W^{1,1}$. It is, however, Hölder continuous (i.e. in a fractional Sobolev space), a.e. approximately differentiable, equals identity on the cube's boundary (and thus is sense-preserving), is measure-preserving (in particular has Lusin's property), and its approximate Jacobian determinant is equal to -1 a.e. Moreover, it is a uniform limit of measure preserving diffeomorphisms.