Some regularity properties of surfaces having mean curvature in L^p

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The talk is based on the joint work with Ulrich Menne.

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We study singular surfaces (viz. 2-dimensional integral varifolds) in $\mathbf{R}^n$ satisfying additional hypotheses on the generalised mean curvature. A classical result [Almgren, unpublished (1965)] states that the class of all m-dimensional integral varifolds in $\mathbf{R}^n$ having locally uniformly bounded mass and first variation is compact. This makes varifolds a natural object of study in the calculus of variations. It has also been long known [Allard, Ann. of Math. (1972)] that if the generalised mean curvature of a varifold V is in L^p for some $p > m$, then a relatively open and dense subset of the support of V is an embedded $C^{1,\alpha}$ manifold ($\alpha = 1 - m/p$). More recently [Wickramasekera, Ann. of Math. (2014)] it was proven that if a codimension 1 integral varifold is stationary (mean curvature is zero) and stable (second variation is nonnegative) and no tangent cone consists of three or more half-hyperplanes meeting along a common codimension 2 vector spaces, then the support of the varifold is a smooth hypersurface outside a set of codimension at least 8 ($\dim V - 7$).

In all the regularity results concerning varifolds a crucial role is played by various estimates on the *tilt-excess*, i.e. mean deviation of the tangent plane to a given plane measured in L^2. They turned out to be useful also for proving perpendicularity of the mean curvature vector for integral varifolds [Brakke, Math. Notes, (1978)], locality of mean curvature [Schätzle, J. Differential Geom.(2009)] as well as C^2-rectifiability of integral varifolds whose first variation is a Radon measure [Menne, J. Geom. Anal. (2013)]. Besides being used as an intermediate step in various proofs, the notion of tilt-excess decay serves itself as weak measure of regularity.

Optimal decay rates are known for m-dimensional varifolds having mean curvature in L^p for the cases $m > 2$ and $p \geq 1$ [Menne, Arch. Ration. Mech. Anal. (2012)] or $p > m$ and $p \geq 2$ [Schätzle, Ann. Sc. Norm. Super. Pisa Cl. Sci. (2004)]. In a joint work with Menne, we resolve the only remaining case, i.e. $m = 2$, $p = 1$ and prove sharpness of our result.