A remark on the Lavrentiew gap phenomenon for harmonic maps into spheres

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We prove that for each positive integer N the set of smooth, zero degree maps $\psi : \mathbb{S}^2 \to \mathbb{S}^2$ which have the following three properties:

(i) there is a unique *minimizing harmonic map* $u : \mathbb{B}^3 \to S^2$ which satisfies the prescribed boundary condition $u\,|_{\partial\mathbb{B}^3} = \psi$;

(ii) this map u has at least N singular points in $\mathbb{B}^3$;

(iii) the Lavrentiev gap phenomenon holds for ψ, i.e.,

$$\min_{W^{1,2}_{\psi}(\mathbb{B}^3,\mathbb{S}^2)} E(u) < \inf_{W^{1,2}_{\psi}(\mathbb{B}^3,\mathbb{S}^2)\cap C^0(\overline{\mathbb{B}}^3)} E(u),$$

where $W^{1,2}_{\psi}(\mathbb{B}^3,\mathbb{S}^2) = \{v \in W^{1,2}(\mathbb{B}^3,\mathbb{S}^2) : v\,|_{\partial\mathbb{B}^3} = \psi \text{ in the trace sense}\}$,

is dense in the set of all smooth zero degree maps $\phi : \mathbb{S}^2 \to \mathbb{S}^2$ endowed with the $H^{1/2}$–topology.