A theorem on Radon measures motivated by vortex sheets

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During the talk I will present a condition which allows to state whether measures supported on spiral curves are locally in $H^{-1}(\mathbb{R}^2)$. The measures we consider correspond to vortex sheets i.e. describe the vorticity of irregular flows. Spiral vortex sheets and their evolution were first observed and analysed by physicists in 1930s, but it is still not known whether they are solutions to the 2d Euler equation. The question whether the spirals are elements of H^{-1} was motivated by the fact, that the existence of vortex solution of the Euler equation was proved [1] under the assumption that the initial vorticity is a compactly supported Radon measure belonging to H^{-1}.

The theorem I will present applies to a broader class of measures than measures supported on spiral curves – namely to all compactly supported Radon measures with prescribed (in a certain way) relation between measure of a ball centred at the origin and its radius. If time permits I will also show how to use our methods to get a new proof of the fact that the Morrey space of measures embeds compactly in H^{-1} (the embedding theorem was first proved in [2]).

References

[1] J. M. Delort *Existence de nappes de tourbillon en dimension deux.*, J. Amer. Math. Soc. 4 553–586, (1991).

[2] M. C. Lopes Filho, H. J. Nussenzveig Lopes, S. Schochet, *A criterion for the equivalence of the Birkhoff-Rott and Euler descriptions of vortex sheet evolution.*, Trans. Amer. Math. Soc. 359 4125–4142, (2007).