Morse–Smale theory on manifolds with boundary: the case of m-functions

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By an m-function we mean a function on a manifold with boundary having only non-degenerate critical points, no critical points near the boundary and such that its restriction to the boundary has only non-degenerate critical points. This is one of possible generalizations of Morse functions to the case of manifolds with boundary, the other one assumes that on each component of the boundary the function is constant. The study of such functions was started by Jankowski and Rubinsztein [4], Braess [2]. Later the Morse - Smale theory for such type of functions was developed in [3]. The interest for this theory was revived by applications to monopole theory [5]. In [1] the results of [4, 2, 3] were rediscovered and detailed analytical proofs were provided.

I will discuss the aspects of the theory of m-functions which seem to be most interesting nowadays. In particular, I will show that the results of [3] give the Morse inequalities for m-functions improving those obtained in [6] by taking into account some mysterious homology operations. I will discuss also another basic method introduced in [3] of replacing a critical point in the interior by two critical points on the boundary. These ideas give the base for the calculations of minimal numbers of critical points of m-functions for simply connected manifolds of dimension at least 6, which are the main results of [3].

References

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