Conformality of a differential

Wojciech Kozłowski

Uniwersytet Łódzki, Poland
wojciech@math.uni.lodz.pl

Session: 12. Geometry and Topology of Manifolds

Consider a Riemannian manifold (M, g). Let $\pi\colon TM \to M$ be a natural projection. The Levi-Civita connection ∇ of g, gives a natural splitting $T(TM) = H \oplus V$ of the second tangent bundle $\pi_*\colon T(TM) \to TM$, where the vertical distribution V is the kernel of π_*, and the horizontal distribution H is the kernel of, so called, connection map K. We say that a Riemannian metric h on TM is *natural* if $\pi\colon (TM, h) \to (M, g)$ is a Riemannian submersion (with respect to the splitting $T(TM) = H \oplus V$). In the talk we introduce some special class of natural metrics, called Cheeger-Gromoll type metrics. Next we give an answer to the following problem:

Let $\varphi\colon (M, g) \to (M', g')$ be a smooth map between Riemannian manifolds. Equip tangent bundles TM and TM' with Cheeger-Gromoll type metrics h and h', respectively. When $\Phi = \varphi_*\colon (TM, h) \to (TM', h')$ is conformal?

Interesting enough, there is an essential difference between the cases $\dim M = 2$ and $\dim M \geq 3$. We show that in the second case Φ is conformal if and only if φ is a homothety and totally geodesic immersion and some special relations between h and h' hold. In this case Φ is also a homothety with the same dilatation as φ. However, in the first case it may happen that Φ is conformal, although φ is not a totally geodesic immersion. Then Φ is no longer a homothety. An example of such a map is given.