Global (weak) solution of the chemotaxis-Navier-Stokes equations with non-homogeneous boundary conditions and logistic growth

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In biology, the behavior of a bacterial suspension in an incompressible fluid drop is modeled by the chemotaxis-Navier-Stokes equations. In this lecture, we introduce an exchange of oxygen between the drop and its environment and an additionally logistic growth of the bacteria population. A prototype system is given by

$$\begin{cases} n_t + u \cdot \nabla n = \Delta n - \nabla \cdot (n \nabla c) + n - n^2, & x \in \Omega,\ t > 0, \\ c_t + u \cdot \nabla c = \Delta c - nc, & x \in \Omega,\ t > 0, \\ u_t = \Delta u + u \cdot \nabla u + \nabla P - n \nabla \phi, & x \in \Omega,\ t > 0, \\ \nabla \cdot u = 0, & x \in \Omega,\ t > 0 \end{cases}$$

in conjunction with the initial data $(n, c, u)(\cdot, 0) = (n_0, c_0, u_0)$ and the boundary conditions

$$\frac{\partial c}{\partial \nu} = 1 - c,\ \frac{\partial n}{\partial \nu} = n \frac{\partial}{\partial \nu} c,\ u = 0, \quad x \in \partial\Omega,\ t > 0.$$

Here, the fluid drop is described by $\Omega \subset \mathbb{R}^N$ being a bounded convex domain with smooth boundary. Moreover, ϕ is a given smooth gravitational potential.

Requiring sufficiently smooth initial data, the lecture gives an outline of the proofs of the existence of

- a global bounded classical solution for $N = 2$ and
- a global weak solution for $N = 3$.