On a Willmore-Helfrich L^2-flow of open curves in $\mathbb{R}^n$

Anna Dall'Acqua

Ulm University, Germany
anna.dallacqua@uni-ulm.de

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In the Bernoulli model of an elastic rod described by a curve $f : I \to \mathbb{R}^n$, $n \geq 2$, the elastic energy is given by

$$\mathcal{E}(f) = \int_I |\vec{\kappa}|^2 \, ds \,,$$

where $\vec{\kappa}$ is the curvature and s is the arc-length parameter. A natural approach to get to minimisers is to study the associated steepest descent flow. Due to the behavior of the energy under scaling, one soon notices that the growth of length of the curve should be penalized (alternatively one could impose that the length does not change).

Motivated by the model of an elastic rod we study the evolution of open curves with fixed end-points, subject to natural boundary conditions and moving in time so that the energy decreases. We show that, if the initial datum is smooth enough, the solution exists globally in time and subconverges to a critical point. Our results apply also to the more general situation where the energy functional contains a so-called spontaneous curvature.

References

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