Rate of convergence to separable solutions of the fast diffusion equation

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We consider the Cauchy problem

$$\begin{cases} u_\tau = \Delta(u^m), & x \in \mathbb{R}^n,\ \tau \in (0,T), \\ u(x,0) = u_0(x) \geq 0, & x \in \mathbb{R}^n, \end{cases}$$

where $n \geq 3$, $T > 0$ and $0 < m < 1$. It is known that for $m < m_c := (n-2)/n$ all solutions with initial data satisfying

$$u_0(x) = O\left(|x|^{-\frac{2}{1-m}}\right) \qquad \text{as } |x| \to \infty,$$

extinguish in finite time. We shall consider solutions which vanish at $\tau = T$ and study their behaviour near $\tau = T$.

The function

$$u(x,\tau) := \big((1-m)(T-\tau)\big)^{\frac{1}{1-m}} \varphi^{\frac{1}{m}}(x) \tag{1}$$

is a solution of the fast diffusion equation $u_\tau = \Delta(u^m)$ if φ satisfies

$$\Delta\varphi + \varphi^p = 0, \qquad x \in \mathbb{R}^n, \qquad p := \frac{1}{m}\,.$$

We call a nontrivial solution of the form (1) separable. We shall show that separable solutions are stable in a suitable sense if

$$n > 10, \qquad 0 < m < \frac{(n-2)(n-10)}{(n-2)^2 - 4n + 8\sqrt{n-1}}\,.$$

We also find optimal rates of convergence to separable solutions.