Exponential Equilibration in a Gradient Flow System Modelling Chemotaxis

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We consider the following variant of the Keller-Segel system for chemotaxis,

$$\partial_t \rho = \Delta(\rho^2) + \nabla \cdot \big(\rho[W + \varepsilon\phi(c)]\big),$$
$$\partial_t c = \Delta c - \kappa c - \varepsilon\rho\phi'(c),$$

with nonlinear mobility of the bacteria and a (possibly non-linear) chemotactic sensitivity ϕ. Solutions (ρ, c) to this system of equations constitute a gradient flow in the coupled Wasserstein–L^2–metric. The flow's driving functional has no useful convexity properties with respect to that metric, but it is the ε-perturbation of an entropy functional that is λ-convex, with a positive λ if the external potential W is sufficiently confining. With the help of this special structure, we prove the global-in-time existence of weak solutions, and their convergence to equilibrium, exponentially fast in time.