Lower bounds on $\dot{H}^s$ norm of blow up solutions of the 3D Navier–Stokes equations

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The well-known observation due to Leray gives the following lower bound

$$\|u(T-t)\|_{L^p} \geq c_p t^{-\frac{p-3}{2p}}$$

for the L^p norm of the solution of the Navier–Stokes equations in 3D that blows up in finite time T. From this estimate and the Sobolev embedding theorem one can deduce lower bounds on the norms of blow up solutions in the homogeneous Sobolev spaces $\dot{H}^s$ for $1/2 < s < 3/2$:

$$\|u(T-t)\|_{\dot{H}^s} \geq c_p t^{-\frac{2s-1}{4}}.$$

The case $s > 3/2$ has to be treated separately. In my talk I will prove optimal results for blow up in $\dot{H}^s$ spaces for $3/2 < s < 5/2$, and sketch known results for $s > 5/2$. Some open problems will be stated as well. This talk will be based on a joint work with James Robinson and Ricardo Silva.