On rings with finite number of orbits

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The talk is based on the joint work with Jan Krempa

Session: 14. Group Rings and Related Topics

Let R be an associative unital ring with the unit group $U(R)$. The additive group of R is denoted by R^+. In [*Rings with finitely many orbits under the regular action,* Lecture Notes in Pure and Applied Mathematics 236, Dekker, New York 2004, 343–347], Yasuyuki Hirano concentrated on the left regular group action of $U(R)$ on R^+ defined by $a \rightharpoonup x = ax$ for all $a \in U(R)$, $x \in R$. The main result of this paper asserted the equivalence of the following statements: (i) R has only a finite number of orbits under the left regular action of $U(R)$ on R^+; (ii) R has only a finite number of left ideals. In the talk we will consider the more general group action of $U(R) \times U(R)$ on R^+ defined by

$$(a, b) \rightharpoonup x = axb^{-1}, \tag{1}$$

for all $a, b \in U(R)$, $x \in R$. The action (1) induces in a natural way an action of the group $U(R) \times U(R)$ on the set of left (respectively, principal left) ideals of R and of ideals of R, however the action on the latter set is trivial. Orbits under the action (1) are called simply U-orbits. We introduce the following properties: *FNE* - R has only a finite number of U-orbits of elements; *FNPLI* - R has only a finite number of U-orbits of principal left ideals; *FNLI* - R has only a finite number of U-orbits of left ideals; *FNI* - R has only a finite number of U-orbits of ideals (R has only a finite number of ideals). We can directly verify the following connections between the above properties:

$$FNE \Rightarrow FNPLI \Rightarrow FNI \quad \text{and} \quad FNLI \Rightarrow FNPLI \Rightarrow FNI. \tag{2}$$

Since for any division ring D and any positive integer n, the $n \times n$ matrix ring $M_n(D)$ has exactly $n + 1$ U-orbits both of elements and of left ideals, it follows that *every semisimple artinian ring satisfies all the properties listed in Formula (2).* In the talk we will discuss two questions. One of them is: *Under which conditions does a left and/or right artinian ring satisfy FNE or a similar property?* The other is: *Must every ring satisfying FNE or a similar property be left and/or right artinian, or at least semiprimary?* Continuuing our discussion we will concentrate on *group rings with finite number of orbits.*

This talk is an outgrowth of our joint paper with Jan Krempa [*On rings with finite number of orbits,* Publicacions Matemàtiques 58 (2014), 233–249].