

Sylow numbers of finite groups

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Let G be a finite group and let $n_p(G)$ denote the number of Sylow p -subgroups of G . The set $sn(G)$ of all Sylow numbers $n_p(G)$ is called Sylow numbers of G . There are many results concerning the structure of groups with given Sylow numbers, especially about the solvability of groups with given Sylow numbers.

F. Luca showed in [1] based on a result of J.Zhang [2] that G is solvable provided $|sn(G)| = 2$, $sn(G) = \{1, a, b\}$ or $sn(G) = \{q^x, a, b\}$, where q is a prime number and either $\gcd(a, b) = 1$ or $q \nmid ab$. N. Chigira however showed that Zhang's result is not valid for all groups [3]. 2012 A. Moreto gave a complete but different proof of the first part of Luca's claim [4].

In the first part of the talk the remaining parts of Luca's result are proved [5].

In the second part the question posed by A. Moreto in [4] whether Sylow numbers are determined by the character table of a group is considered. It is shown that this is the case when G is supersolvable (this reports on joint work with W. Kimmerle).

References

- [1] F. Luca, *Groups with two Sylow numbers are solvable*, Arch. Math. 71, 1998, 95–96.
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- [4] A. Moreto, *Groups with two Sylow numbers are the product of two nilpotent Hall subgroups*, Arch. Math 99, 2012, 301–304.
- [5] I. Köster, *Finite groups with Sylow numbers $\{q^x, a, b\}$* , Stuttgarter Mathematische Berichte 2014/10
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