Separability of Free Groups and Surface Groups

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The word problem for a given finitely generated group is the problem of telling whether a word in the generators represents the identity element. For any finitely presented group, this problem has an easy part: if the word is trivial, then it follows from the given finitely many relations; hence it is possible to algorithmically list all trivial words. Thus, in a group with unsolvable word problem, it is impossible to algorithmically list the non-trivial words.

For a finitely presented group, it is easy to list all actions on finite sets: for a proposed action of the generators, just check whether the relations hold. Hence, one can algorithmically list all finite quotients of a finitely presented group. This provides an obvious way of listing some non-trivial words: put down those, that represent a non-trivial element in some finite quotient. A group where this algorithm eventually finds each non-trivial word is called residually finite. Residually finite groups have an obvious solution to the word problem.

The conjugacy problem of telling which words represent conjugate elements allows for a similar treatment. In any finitely presented group, it is algorithmically easy to list all pairs of words representing conjugate elements. The hard part is to list the pairs of words representing non-conjugate elements. A group is called conjugacy separable, if any two non-conjugate elements stay non-conjugate in some finite quotient. Thus, finitely presented conjugacy separable groups admit an obvious solution to the conjugacy problem.

Other classical algorithmic problems can be treated analogously. Each leads to a corresponding notion of separability. The problem of telling whether two finitely generated subgroups are conjugate gives rise to the notion of subgroup conjugacy separability. A group is subgroup conjugacy separable if any two non-conjugate finitely generated subgroups have non-conjugate images in some finite quotient. We show that finitely generated free groups and fundamental groups of closed oriented surfaces are subgroup conjugacy separable.