# Combinatorial constructions in Smooth Ergodic Theory

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*Parts of the talk are based on the joint work with Roland Gunesch.*

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In my talk I will present a multitude of new constructions of diffeomorphisms with various specific ergodic, topological and spectral properties. These are based on the "Conjugation by approximation"-method developed by D.V. Anosov and A. Katok in [1]. In fact on every smooth compact connected manifold of dimension $m \geq 2$ admitting a non-trivial circle action $\mathcal{S} = \{S_t\}_{t \in \mathbb{S}^1}$ preserving a smooth volume $\nu$ this method enables the construction of smooth diffeomorphisms with particular ergodic properties or non-standard smooth realizations of measure preserving systems. Moreover, one can deduce results on the genericity of designed properties.

One main topic is the construction of weak mixing diffeomorphisms preserving a measurable Riemannian metric in the restricted space $\mathcal{A}_\alpha(M) = \overline{\{h \circ S_\alpha \circ h^{-1} \ : h \in \mathrm{Diff}^\infty(M, \nu)\}}^{C^\infty}$ for a given Liouvillean number $\alpha \in \mathbb{S}^1$. So we design maps with predetermined rotation number. In addition, we examine the existence of such diffeomorphisms on the $m$-dimensional torus $\mathbb{T}^m$, $m \geq 2$, with prescribed number of ergodic invariant measures, in particular uniquely ergodic ones.

Furthermore, we start to examine the problem of uniformly rigid and simultaneously weak mixing maps, which is an up-to-date research topic in measurable as well as topological dynamics, in the smooth and beyond that even in the real-analytic category: Under sufficient conditions on the growth rate of the rigidity sequence we are able to construct uniformly rigid and weak mixing real-analytic as well as $C^\infty$-diffeomorphisms on $\mathbb{T}^2$.

## References

[1] D. V. Anosov, A. Katok, *New examples in smooth ergodic theory. Ergodic diffeomorphisms*, Trudy Moskov. Mat. Obsc. 23, 1970, 3–36.