On specification-like properties and simplices of invariant measures of dynamical systems

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The talk is based on the joint work with Dominik Kwietniak.

Session: 16. Ergodic Theory and Dynamical Systems

We study invariant measures of a dynamical system given by a continuous map on a compact metric space. Ergodic measures are extreme points of the simplex of all invariant measures and every invariant measure is a generalized affine combination of ergodic ones. Hence, the set of invariant measures is a Choquet simplex. The Poulsen simplex is an universal object among all Choquet simplices. It is characterized as the unique infinite-dimensional Choquet simplex in which extreme points are dense. This brings us to the question: *When the set of invariant measures of a dynamical system is the Poulsen simplex?* This problem has been considered by many authors. Sigmund proved (among many others things) that it is enough to assume that system has the periodic specification property. Examples of such systems include mixing SFT's and sofic shifts, mixing axiom A diffeomorphisms, mixing continuous interval (graph) maps, and geodesic flows on manifolds with negative curvature. Many authors have weakened the specification property to apply similar techniques to a wider range of examples. An (incomplete) list contains such names as Climenhaga, Coudène, Dateyama, Gelfert, Hofbauer, Oliveira, Pfister, Varandas and Yamamoto (and their co-authors). Our work is very much in the spirit of these developments. We define yet another specification-like property and construct examples to examine limitations of various approaches. The main result describes properties of the simplex of invariant measures under our specification-like assumption.

It turns out that the notion of Besicovitch pseudometric is a very useful tool in these investigations. As a main technical tool we introduce the theory of pseudorbits with respect to the Besicovitch pseudometric and measures generated by them. Our version extends the Sigmund theorem and can be applied to examples lying beyond the scope of previously known results.