Representation of Markov chains by random maps: existence and regularity conditions

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Amongst the main concerns of Dynamics one wants to decide whether asymptotic states are robust under random perturbations. Considering the iteration of $f\colon M \to M$, such randomness are represented by a family $\{p_\varepsilon(\,\cdot\,|x)\}$ of Borel probability measures, such that every $p_\varepsilon(\,\cdot\,|x)$ is supported inside the ε-neighbourhood of $f(x)$. Alternatively, the orbit is given by the iteration $x_j = g_j \circ \cdots \circ g_1(x_0)$, where each measurable g_j is picked at random ε-close from the original map f. Endowing the collection of maps $\{g_j\}$ with a probability distribution ν_ε, we say that the sequence of random maps is a representation of that Markov chain if for every Borel subset U, $p_\varepsilon(U|x) = \nu_\varepsilon(\{g : g(x) \in U\})$. In this talk we systematically investigate the problem of representing Markov chains by families of random maps, and which regularity of these maps can be achieved depending on the properties of the probability measures. Our key idea is to use techniques from optimal transport to select optimal such maps. From this scheme, we not only deduce the representation by measurable and continuous random maps, but also obtain conditions for the to construct random diffeomorphisms from a given Markov chain. This is a joint work with Jost, and Kell.