Counting Markov partitions of toral automorphisms using Sturmian words

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For an automorphism of two the dimensional torus by a *Berg partition* we name a Markov partition into two rectangles (or parallelograms depending on coordinates). It is shown that there finitely many Berg partitions. Moreover coding of the chosen eigendirection of the automorphism by any Berg partition gives the same Sturmian sequence. Those Sturmian sequences exhibit existence of pattern structure of palindromes (which is relatively easy to examine geometrically and quite difficult to prove in terms of combinatorics). Using this structure we are able to show that there exactly $\mathrm{ENT}\left(\frac{a+b+c+d}{2}\right)$ Berg partitions with a transition matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

Moreover there are $2(p_1 + p_2 + \ldots + p_n)$ different transition matrices where $(p_1, p_2, \ldots, p_n)$ is a period of the continued fraction expansion of a slope of the principal eigendirection.