Discrete maximal functions in higher dimensions

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Let $T_1, \ldots, T_d \colon X \to X$ be a family of commuting invertible and measure preserving mappings on $(X, \mathcal{B}, \mu)$. Let

$$\mathcal{P} = \big(\mathcal{P}_1, \ldots, \mathcal{P}_d\big) : \mathbb{Z}^k \to \mathbb{Z}^d$$

be a mapping such that each $\mathcal{P}_j$ is an integer-valued polynomial on $\mathbb{Z}^k$ with $\mathcal{P}_j(0) = 0$. We present a higher dimensional counterpart of Bourgain's pointwise ergodic theorem along $\mathcal{P}$. We achieve this by proving variational estimates V_r on $L^p(X, \mu)$ for $p > 1$ and $r > \max\{p, p/(p-1)\}$ for an averaging operator

$$M_N f(x) = \frac{1}{N^k} \sum_{y_1=1}^{N} \cdots \sum_{y_k=1}^{N} f\big(T_1^{\mathcal{P}_1(n)} \cdots T_d^{\mathcal{P}_d(n)} x\big).$$

Moreover, we obtain the estimates which are uniform in the coefficients of $\mathcal{P}$.