Explicit representations of spaces of smooth functions or distributions by sequence spaces

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The Valdivia-Vogt structure table

$$\begin{array}{ccccccccccccccc} \mathcal{D} & \subset & \mathcal{S} & \subset & \mathcal{D}_{L^p} & \subset & \dot{\mathcal{B}} & \subset & \mathcal{D}_{L^\infty} & \subset & \mathcal{O}_C & \subset & \mathcal{O}_M & \subset & \mathcal{E}=\mathcal{C}^\infty \\ \wr\| & & \wr\| & & \wr\| & & \wr\| & & \wr\| & & \wr\| & & \wr\| & & \wr\| \\ \mathbb{C}^{(\mathbb{N})}\widehat{\otimes}_\iota s & \subset & s\widehat{\otimes}s & \subset & \ell^p\widehat{\otimes}s & \subset & c_0\widehat{\otimes}s & \subset & \ell^\infty\widehat{\otimes}s & \subset & s'\widehat{\otimes}_\iota s & \subset & s'\widehat{\otimes}_\pi s & \subset & \mathbb{C}^{\mathbb{N}}\widehat{\otimes}s \end{array}$$

presented in [3] contains the most prominent spaces of smooth functions occurring in the theory of distributions together with their sequence-space representations. Analogously its "dual version"

$$\begin{array}{ccccccccccccc} \mathcal{E}' & \subset & \mathcal{S}' & \subset & \mathcal{D}'_{L^p} & \subset & \mathcal{D}'_{L^\infty} & \subset & \mathcal{O}'_M & \subset & \mathcal{O}'_C & \subset & \mathcal{D}' \\ \wr\| & & \wr\| & & \wr\| & & \wr\| & & \wr\| & & \wr\| & & \wr\| \\ \mathbb{C}^{(\mathbb{N})}\widehat{\otimes}s' & \subset & s'\widehat{\otimes}s' & \subset & \ell^p\widehat{\otimes}s' & \subset & \ell^\infty\widehat{\otimes}s & \subset & s\widehat{\otimes}_\iota s' & \subset & s\widehat{\otimes}_\pi s' & \subset & \mathbb{C}^{\mathbb{N}}\widehat{\otimes}s' \end{array}$$

contains the most prominent spaces of distributions together with their sequence space representations.
In [1] the existence of an isomorphism $\Phi\colon \mathcal{E}\to\mathbb{C}^{\mathbb{N}}\widehat{\otimes}s$ such that every restriction to any other space in the structure table provides an isomorphism between this space and its sequence space representation is shown.
We provide an explicit isomorphism between the spaces $\mathcal{E}(\mathbb{R})=\mathcal{C}^\infty(\mathbb{R})$ and $\mathbb{C}^{\mathbb{N}}\widehat{\otimes}s$ as well as an explicit isomorphism between the spaces $\mathcal{D}'(\mathbb{R})$ and $\mathbb{C}^{\mathbb{N}}\widehat{\otimes}s'$ which allow us to interpret the tables above as commutative diagrams. We use these isomorphisms to construct a common basis for the spaces of smooth functions and one for the spaces of distributions in the tables except $\mathcal{D}_{L^\infty}$ and $\mathcal{D}'_{L^\infty}$, those spaces being non-separable.

References

[1] C. Bargetz, *Commutativity of the Valdivia-Vogt table of sequence space representations of spaces of smooth functions*, Math. Nachr. 287(1): 10–22, 2013.

[2] C. Bargetz, *Explicit representations of spaces of smooth functions and distributions*, Preprint, 2013.

[3] N. Ortner and P. Wagner, *Explicit representations of L. Schwartz' spaces $\mathcal{D}_{L^p}$ and $\mathcal{D}'_{L^p}$ by the sequence spaces $s\widehat{\otimes}\ell^p$ and $s'\widehat{\otimes}\ell^p$, respectively, for* $1<p<\infty$, J. Math. Anal. Appl., 404(1): 1–10, .