Necessary and sufficient conditions for well-posedness of p-evolution equations

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For $p \geq 2$ we consider, in $[0,T] \times \mathbb{R}$, the p-evolution operator P of the form

$$P(t,x,D_t,D_x) = D_t + a_p(t)D_x^p + \sum_{j=0}^{p-1} a_j(t,x)D_x^j,$$

where $a_p \in C([0,T];\mathbb{R})$ and $a_j \in C([0,T];\mathcal{B}^\infty)$ for $0 \leq j \leq p-1$.

We look for necessary and sufficient conditions for well-posedness in H^∞ of the associated Cauchy problem.

The assumption that a_p is real valued means that the principal symbol, in the sense of Petrowski, has the real characteristic $\tau = -a_p(t)\xi^p$ and is due to the Lax-Mizohata Theorem. Many results of well-posedness of the Cauchy problem are known when also the other coefficients a_j, for $0 \leq j \leq p-1$, are real. When a_j are complex valued W. Ichinose proved, in the case $p = 2$, that some decay condition on $\operatorname{Im} a_{p-1} = \operatorname{Im} a_1$ is necessary and sufficient for well-posedness in H^∞.

In [1] we look for sufficient conditions for well-posedness in H^∞, obtaining a set of decay conditions, as $x \to +\infty$, on $\operatorname{Im} D_x^\beta a_j$, for $j \leq p-1$ and $[\beta/2] \leq j-1$.

These results have been extended to the case of p-evolution equations of higher order in [2] and to semi-linear 3-evolution equations in [3].

Then a necessary condition for well-posedness of the Cauchy problem in H^∞ has been proved in [4], generalizing to the case $p \geq 2$ the necessary condition given by Ichinose for $p = 2$.

References

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[4] A. Ascanelli, C. Boiti, L. Zanghirati, *A Necessary condition for H^∞ well-posedness of linear p-evolution equations*, in preparation.