Commutative subalgebras of the algebra of smooth operators

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The significance of the space s of rapidly decreasing sequences in the theory of nuclear Fréchet spaces cannot be overestimated. In fact, by the Kōmura-Kōmura theorem, a Fréchet space is nuclear if and only if it is isomorphic to some closed subspace of $s^{\mathbb{N}}$. Moreover, s is isomorphic as a Fréchet space to many classical spaces of smooth functions, for example it is isomorphic to the Schwartz space $S(\mathbb{R})$ of smooth rapidly decreasing functions on the real line.

In this talk we deal with some noncommutative analogue of the algebra s, namely with the Fréchet *-algebra $\mathcal{L}(s', s)$ of so-called smooth operators. We focus on classification and characterization of closed commutative *-subalgebras of $\mathcal{L}(s', s)$. In particular, we show a surprising fact that a closed commutative *-subalgebra of $\mathcal{L}(s', s)$ is isomorphic as a Fréchet *-algebra to some closed *-subalgebra of s if and only if it is isomorphic as a Fréchet space to some complemented subspace of s. Moreover, we provide an example of a closed commutative *-subalgebra of $\mathcal{L}(s', s)$ which is not embedded isomorphically into s as a closed *-subalgebra.