Vector-valued Fourier hyperfunctions

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Scalar-valued Fourier hyperfunctions were introduced by Kawai [1] in 1970 as a generalization of the sheaf of hyperfunctions which was introduced by Sato [2] ([3]). He constructed them as a flabby sheaf on the radial compactification of $\mathbb{R}^n$ and it turned out that the global sections are stable under Fourier transformation. Vector-valued counterparts for the theory of (Fourier) hyperfunctions were developed, at first, with values in Fréchet spaces and in 2008 Domański and Langenbruch [4] not only extended the theory of hyperfunctions far beyond the class of Fréchet spaces by using new results on splitting theory for PLS-spaces, but they also found natural limits of this kind of theory.
My talk summarizes results of my thesis [5] and is dedicated to the development of the theory of Fourier hyperfunctions in one variable with values in a non-necessarily metrizable locally convex space E. Moreover, necessary and sufficient conditions are described such that a reasonable theory of E-valued Fourier hyperfunctions exists. In particular, if E is an ultrabornological PLS-space, such a theory is possible if and only if E satisfies the so-called property (PA). It turns out that the vector-valued Fourier hyperfunctions can be realized as the sheaf generated by equivalence classes of certain compactly supported E-valued functionals and interpreted as boundary values of slowly increasing holomorphic functions.

References

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