Interpolation of holomorphic functions and surjectivity of Euler type partial differential operators

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The talk is based on the joint work with Pawel Domański (AMU Poznań)

Session: 17. Functional Analysis: relations to Complex Analysis and PDE

We discuss multipliers on real analytic functions, i.e. continuous linear mappings $M : \mathscr{A}(\mathbb{R}^d) \to \mathscr{A}(\mathbb{R}^d)$ such that all monomials are eigenvectors, that is, $M(\xi^\alpha)(x) = m_\alpha x^\alpha$ for any $\alpha \in \mathbb{N}^d$. Classical examples are given by Euler type operators (of finite or infinite order) $P(\theta) := \sum_\alpha c_\alpha \theta^\alpha$ where $\theta^\alpha := \prod_{j\le d} \theta_j^{\alpha_j}$ for $\theta_j := x_j \partial/\partial x_j$. Hence we are dealing also with differential operators of finite or infinite order with polynomial coefficients. By the First Representation Theorem for multipliers (proved jointly with D. Vogt (Wuppertal)) any multiplier M is given by a (unique) analytic functional $T \in \mathscr{A}(\mathbb{R}^d)'$ as follows: $M(g)(y) = M_T(g)(y) := \langle {}_xT \, , \, g(yx)\rangle, g \in \mathscr{A}(\mathbb{R}^d)$, where xy is defined by coordinatewise multiplication. Moreover, the multiplier sequence $(m_\alpha)_{\alpha\in\mathbb{N}^d}$ is equal to the moment sequence $(\langle T, x^\alpha\rangle)_{\alpha\in\mathbb{N}^d}$. Moment sequences of analytic functionals can be characterized by interpolation of certain holomorphic functions. This can be used to study invertibility of Euler type operators $P(\theta)$ on multiplier complemented subspaces of real analytic functions. The necessary and the sufficient criterion for invertibility is closely related to the half plane property for the principal part P_m, namely that $P_m(z) \neq 0$ if $\Re(z_j) > 0$ for any j. Many examples are provided.